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Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

John Borthwick Dale, M.A., Assistant Professor of Mathematics, King's College, London; "Myosotis," New Malden, Surrey;
Rev. William Charles Eppstein, Reading School, Berks;
Walter Nuttall, B.A., Hallfold School, Whitworth, Rochdale, and
William Edward Rolston, Solar Physics Observatory, South Kensington, S.W.;

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:

Edward Barlow, Harewood, Andover, Hants (proposed by Major P. A. MacMahon);
Otto J. Klotz, Astronomer, Department of the Interior, Ottawa, Canada (proposed by W. H. M. Christie);
Major C. Leigh-Lye, 15 Mayfield Avenue, Chiswick, W. (proposed by A. C. D. Crommelin).

The following were proposed by the Council as Associates of the Society:

Henri Deslandres, D.-ès-Sc., F.R.A.S., Observatoire d'Astronomie Physique, Meudon, S.-et-O., France;
C. D. Perrine, Lick Observatory, Mount Hamilton, California, U.S.A.;
George W. Ritchey, Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.

Seventy presents were announced as having been received since the last meeting, including, amongst others :

Annales de l'Observatoire de Meudon—Atlas de Photographies Solaires, presented by M. Janssen ; Observatoire d'Alger, Catalogue Photographique du Ciel, 3 fascicules ; and Introduction by Ch. Trépied, presented by the Observatory ; G. E. Hale and F. Ellerman (&c.), The Rumford Spectroheliograph of the Yerkes Observatory, and Spectra of Stars of Secchi's fourth type, presented by Professor Hale ; E. B. Frost and W. S. Adams, Radial Velocities of Stars of the *Orion* type, presented by Professor Frost ; S. W. Burnham, Measures of Double Stars, presented by the Author ; Tokyo Observatory, Annales, tomes 2, 3 (with report of Solar Eclipse of 1901), presented by the Observatory.

On the Degree of Accuracy of the new Lunar Theory and on the Final Values of the Mean Motions of the Perigee and Node. By Ernest W. Brown, Sc.D., F.R.S.

In the *Monthly Notices* for 1903 May I gave a general statement of the degree of precision existing between the calculated values of the mean motions of the perigee and node and their observed values. It was added that the final verifications of the higher terms due to solar action were not complete, but an upper limit was given for possible errors. Since that date the calculations for these higher terms have been tested and found to be correct, so that the portions due to solar action are completely finished, and the portion arising from each characteristic is calculated to 0″·01 in the annual motions.

In order to make the comparison complete to this degree of accuracy it was found necessary to undertake an examination into the numerical values of the constants used and into the effects produced by sources other than the direct solar action. This inquiry revealed several differences which had a perceptible effect. It was first necessary to develop a general method of dealing with the effects of planetary and other perturbations on these mean motions which would permit of their being found easily and accurately. Two methods for the solution of this problem were obtained ; they will be published elsewhere,* the numerical results only being given here. It was found necessary to make several changes in the values of these non-solar perturbations as collected in my papers in the *Monthly Notices* for 1897 March and June : four of these caused alterations of about half a second each in the annual motions. First, in multiplying the ratios of the motions to n—ratios which were found correctly from theory—by n (the mean motion), it was found that a

* *Trans. Amer. Math. Soc.* July 1904.

slightly inaccurate value of n had been used. Second, Hansen's value for the planetary action on the perigee was found to be wrong by a quarter of its whole amount. Third, Hill's values for the effects produced by the figure of the earth correspond to an ellipticity rather greater than would seem to be admissible. Fourth, Hansen's and Newcomb's values, deduced from observation, correspond to the epoch 1800, while the values of the constants used in my theory, especially that of the solar eccentricity, correspond to the epoch 1850. After these changes had been made it was found that the agreement between theory and observation was not impaired; in fact, the differences are still within the limits of error arising from doubts as to the correct values to be assigned to certain constants. As far as the lunar theory is concerned, every theoretical result set down here has been calculated or tested afresh by one, or in some cases two methods; the only exception is in a case where little doubt can be entertained and where in any case the effect is very small—namely, the perturbations of the solar radius vector, and these have been taken from Newcomb's *Tables of the Sun*.

There is only one constant whose observed value is so far doubtful as to affect the results by so much as $0''{\cdot}1$—the ellipticity of the earth. For this constant there seem to be really two competing values, one about $1/293$ and the other about $1/297$. As it appears difficult to make a choice between them I give two calculated results corresponding to two values of the ellipticity, the final determinations are as follows:

Annual Mean Motions, Epoch 1850·0.

	Perigee.			Node.		
Calculated (α)	+146	435″·27	±0″·10,	−69	679″·37	∓0″·05;
Calculated (β)	+146	435·11	±0·10,	−69	679·22	∓0·05;
Observed	+146	435·23	... ,	−69	679·45	... ;
C−O (α)	...	+0·04	... ,	...	+0·08	... ;
C−O (β)	...	−0·12	... ,	...	+0·23	... .

(α) corresponds to an ellipticity $1/292{\cdot}9$;
(β) „ „ „ $1/296{\cdot}3$;

the quantities $\pm 0''{\cdot}10$, $\mp 0''{\cdot}05$ being extreme possible errors due to neglect of terms whose orders are higher than those calculated and to the doubt as to the correct value of the mass of *Venus*. The direct and indirect effects due to *Mercury* are nearly equal and opposite, so that the uncertainty as to the mass of this planet does not affect the results. No possible errors due to the use of a limited number of places of decimals have been set down, since nearly every number has been computed to at least one more place than that actually given; the errors from this cause are probably less than $0''{\cdot}05$.

The portions which various parts of the complete disturbing function produce will now be considered in detail. So far as I know, every purely gravitational cause which can give a sensible effect has been considered; the list in No. 7 below will, in any case, permit of any changes or additions to be easily made.

The references are collected at the end of the paper and are referred to by the letters (*a*), (*b*), ...

The adopted values of the constants are for the epoch 1850·0:

Moon.	Sun.
$n = 17\ 325\ 594''$,	$n' = 1\ 295\ 977''\cdot4$;
$e = \cdot054\ 900\ 3$,	$e' = \cdot016\ 772$;
$\gamma = \cdot044\ 887\ 2$,	$\gamma' = 0$;
Parallax $= 3422''\cdot7$,	Parallax $= 8''\cdot80$;
E/M $= 81\cdot5$,	$m'/(\mathrm{E}+\mathrm{M}) = 328\ 243$;

where n, n' are the annual mean motions; E, M, m' the masses of the Earth, Moon, Sun; e, the lunar eccentricity corresponding to Newcomb's coefficient $22659''\cdot58$ for the principal elliptic term (*a*); e', the solar eccentricity (*b*); γ, the sine of half the inclination corresponding to Newcomb's coefficient $18461''\cdot48$ for the principal term in latitude (*a*); the parallaxes are those generally accepted at the present time. No probable change in any of these constants will materially alter the values for the mean motions.

1. *The solar portions.*—These are obtained by using the disturbing function (*a*)

$$(\mathrm{E}+\mathrm{M}+m')\left[\frac{1}{\sqrt{(x-x')^2+(y-y')^2+z^2}}-\frac{xx'+yy'}{r'^3}\right]$$
$$=\frac{n'^2a'^3}{r'^3}\left\{r^2\left(\tfrac{3}{2}\mathrm{S}^2-\tfrac{1}{2}\right)+\frac{r^3}{a'}\left(\tfrac{5}{2}\mathrm{S}^3-\tfrac{3}{2}\mathrm{S}\right)+\frac{r^4}{a'^2}\left(\tfrac{35}{8}\mathrm{S}^4-\tfrac{15}{4}\mathrm{S}^2+\tfrac{3}{8}\right)\right\}$$

where the solar coordinates x', y', z', r' refer to the elliptic motion of the Sun about the Earth with the correction

$$\frac{\mathrm{E}-\mathrm{M}}{\mathrm{E}+\mathrm{M}}\cdot\frac{a}{a'}=a \text{ for } \frac{a}{a'}$$

in the final results—a correction which accounts for the greater part of the motion of E about the centre of mass of E and M. Here S is the cosine of the angle between the radii to the Sun and Moon.

The portions calculated are those with characteristics 1, e^2, γ^2, e'^2, $(a/a')^2$, e^4, $e^2\gamma^2$, γ^4, $e^2e'^2$, $\gamma^2e'^2$. A consideration of the numerical values of other characteristics shows that the sum of the terms which arise from them will be less than $0''\cdot04$ in the case of the perigee and less than $0''\cdot02$ in the case of the node.

The terms of characteristic unity form respectively 71/70, 174/175 of the complete values.

In an earlier paper (*d*) I have given results (extensions of Adams' theorems) which directly connected the parts in the motion of the perigee having characteristics $e^2\gamma^2$, γ^4, $\gamma^2 e'^2$ with those in the motion of the node having characteristics e^4, $e^2\gamma^2$, $e^2e'^2$ respectively. The former set are determined in the theory with the terms in x, y (longitude and parallax), and the latter with the terms in z (latitude), which are of the fifth order with respect to their characteristics. The two sets are calculated in totally different ways, the former with the "homogeneous" equations and the latter with the non-homogeneous equation. There were thus three separate tests, and in all cases the results agreed within two units of the last place of decimals used. These searching tests, involving nearly all the results of the third order, a large proportion of those of the fourth order, and the greater part of the forms of computation for the terms of the fifth order, would seem to warrant a feeling of security in the general accuracy of the whole theory, so far as it has gone.

But these three tests, though they are exact and searching, are on the whole less valuable than an indirect test for general accuracy which arises automatically in every set of terms with a given characteristic. Each such set in the x, y coordinates and every other set in the z coordinate has one (long-period) or two (monthly) terms whose coefficients possess small divisors, and the process of division by these divisors is practically the last step in the determination of the coefficients. Now, the possibility of error in the *details* of numerical calculation has been practically eliminated ; the chief danger arises from the possible omission of a whole set of terms, or the use of a wrong set. But it is precisely the latter forms of error which the test guards against, for such mistakes would almost certainly cause the coefficients to be many times greater than their actual values, and a rough comparison with the results of Delaunay would show the existence of error at once. In fact, the numbers before division are composed of quantities which are large in comparison with their algebraical sum. Thus, the chief cause of the enormous extent of the computations furnishes the most valuable test of their accuracy.

2. *Correction due to using* $\mathrm{E}+\mathrm{M}+m'$ *instead of* m' *as the factor of the disturbing function.*—The additional sensible portion is (*c*)

$$-\frac{\mathrm{E}+\mathrm{M}}{\mathrm{E}+\mathrm{M}+m'}\, n'^2 r'^2 \left(\tfrac{3}{2}\mathrm{S}^2-\tfrac{1}{2}\right).$$

The terms of characteristic unity, due to this function, have been calculated by the method of variation of arbitraries as well as by the method of the theory, and the results agreed in giving $-0''{\cdot}71$ for the perigee and $+0''{\cdot}20$ for the node. Multiplying these by 70/71, 175/174 respectively, which accounts sufficiently

for the terms in higher characteristics, we obtain $-0''\cdot70$, $+0''\cdot20$, agreeing with the estimate which I gave in an earlier note (*e*). No possible alteration in the value attached to the ratio $(E+M)/(E+M+m')$ will change these by so much as $0''\cdot01$.

3. *Correction for the remaining effect of the mass of the Moon.*—The addition to the disturbing function (*c*) is a term

$$\frac{EM}{(E+M)^2}\, n'^2\, \frac{r^4}{a'^2}\left(\tfrac{35}{8}S^4-\tfrac{15}{4}S^2+\tfrac{3}{8}\right).$$

It gives $+0''\cdot02$ for the perigee, $-0''\cdot01$ for the node. The indirect action produced by a corresponding term in the Sun's disturbing function gives $-0''\cdot01$ and $0''\cdot00$ in the respective cases.

4. *Direct planetary action.*—This has been calculated by the method of the theory (for its principal part) and also completely by the variation of arbitrary constants. Using Hansen's values for the masses of the planets, and his expansions of the disturbing functions, only in so far as they depend on the positions of the Earth and planets, it appears that Hansen's result for the perigee is much too small (*f*). Adopting modern values for the masses and Newcomb's expansion for *Venus* (*h*), which has the greatest effect, I find $+2''\cdot66$ for the perigee and $-1''\cdot42$ for the node. Hansen (*f*) gave $+2''\cdot10$ and $-1''\cdot50$. My results include squares of the eccentricities and inclinations of the Moon and planets, and all powers of *m*. The use of these more accurate values for the parts depending on the planets would subtract about $0''\cdot1$ from Hansen's result for the perigee, so that it is $0''\cdot7$ in error. The node is practically unaffected. I have not been able to trace the source of this error in Hansen very far ; it depends, however, on the terms factored by K in Art. 172 of the *Darlegung*, the value of K being correctly found with the adopted values of the masses.

The following table gives the values of the masses adopted here (*b*) with the corresponding motions and the extreme amounts by which the latter can be in error owing to the uncertainty in the mass of *Mercury*, which is doubtful to 50 per cent. of the given mass, and in the mass of *Venus*, which is doubtful to 5 per cent. :

Direct Action of Planets.

	Recip. of Mass.	Perigee.	Node.
Mercury	6,000,000	$+0''\cdot049 \pm \cdot025$	$-0''\cdot016 \mp \cdot008$
Venus	408,000	$+1\cdot966 \pm \cdot098$	$-\cdot819 \mp \cdot041$
Mars	3,093,000	$+\cdot034$	$-\cdot025$
Jupiter	1,047·35	$+\cdot582$	$-\cdot536$
Saturn	3,501· 6	$+\cdot026$	$-\cdot025$
Sum	...	$+2\cdot657 \pm 123$	$-1\cdot421 \mp \cdot049$

The extreme errors, when taken in connection with those produced by the indirect action, are diminished to one-half of these values, as appears in No. 6 below.

5. We have put, in the solar disturbing function after the correction of No. 2 has been made,

$$E+M'+m' = n'^2 a'^3,\ r' = a' + \text{elliptic terms}.$$

The latter value of a' requires a correction, the constant part of which will be denoted by $\delta a'$, on account of the perturbations produced in the Sun's motion by the planets. The addition to the disturbing function for the Moon is

$$-3\frac{\delta a'}{a'}\, n'^2 r^2 (\tfrac{3}{2}S^2 - \tfrac{1}{2}).$$

It is therefore similar to that of No. 2, and the corrections to the perigee and node are respectively

$$-\frac{3\delta a'}{a'} \cdot \frac{0''\cdot 70}{328243}, \quad +\frac{3\delta a'}{a'} \cdot \frac{0''\cdot 20}{328243}.$$

The value of $\delta a'/a'$ for *Mercury* has been obtained from Newcomb (*i*) and for the other planets and for the adopted value of the gravitation constant from his tables of the Sun (*b*). The corrections, with the extreme errors due to uncertainty in the adopted masses of *Mercury* and *Venus*, are given in the following table :

Indirect Planetary Action.

	$\log_{10}(1+\delta a'/a')$.	Perigee.	Node.
Mercury ...	+·000 000 028	$-0''\cdot041 \mp \cdot020$	$+0''\cdot012 \pm \cdot006$
Venus ...	+ 634	$- \cdot934 \mp \cdot047$	$+ \cdot267 \pm \cdot013$
Mars ...	− 12	+ ·018	− ·005
Jupiter ...	− 512	+ ·754	− ·215
Saturn ...	− 24	+ ·036	− ·010
Grav. const. ...	+ 13	− ·019	+ ·005
Sum ...	+·000 000 127	$-0''\cdot186 \mp \cdot067$	$+0''\cdot054 \pm \cdot019$

The action of *Venus* produces a periodic inequality with a coefficient of about 15″. When squares of the action are included, this inequality will give rise to small changes in the mean motions of the perigee and node. These I have examined and found to be absolutely insensible.

6. *Total planetary action.*—It will be noticed that the direct and indirect actions of *Mercury* are opposite in effect and nearly equal, so that any uncertainty as to its mass does not affect the results by so much as 0″·01. The indirect action of *Venus* is about half the direct action with the opposite sign.

Hence, combining the results in Nos. 4, 5, we can subtract the two sets of extreme errors and obtain for the

Total Planetary Action.

Perigee.	Node.
$+2''\cdot48 \pm 0''\cdot06$	$-1''\cdot37 \mp 0''\cdot03$,

the extreme errors arising entirely from doubt as to the mass of *Venus*.

7. *Figure of the Earth effects.*—The part of the disturbing function due to the figure of the earth, which alone can give rise to sensible terms in the mean motions of the perigee and node, is

$$\lambda\left(\tfrac{1}{3}\frac{a^3}{r^3}-\frac{z^2\,a^3}{r^5}\right).$$

Hill (*k*) has made an extended literal investigation by Delaunay's method of the terms arising from this function, but it is open to the objection that the results for the perigee and node are only carried as far as the orders λm^3, λe^2, $\lambda\gamma^2$. It appears from the expression for the perigee, which begins with a term of order λm^0, that slow convergence along powers of m might make a difference of as much as $0''\cdot3$ from Hill's result. I therefore made a new and different computation, using the numerical value of m and carrying the approximations so far as to include all sensible terms of the forms

$$\lambda f(m),\ \lambda e^2 f(m),\ \lambda\gamma^2 f(m),\ \lambda e'^2 f(m).$$

The terms of the form $\lambda f(m)$ were also computed by the method of my theory. It appeared that the apparent slow convergence had very little effect. Hill's results for the perigee and node were $+6''\cdot82$, $-6''\cdot41$ respectively, while mine gave $+6''\cdot86$, $-6''\cdot42$, using Hill's value for λ. The only possible source of error is therefore in the value to be substituted for λ.

The usual method, and probably the best one for the purposes of the lunar theory, is to determine λ by means of the coefficient of the principal term, due to the figure of the Earth, in latitude. This, according to Hill, has a coefficient $-8''\cdot726$, and, excluding the doubt as to λ, it is probably accurate within $0''\cdot003$. Hansen (*f*) finds $-8''\cdot382$ for the observational value of the coefficient. There is a term due to the motion of the ecliptic with the same argument, and with a coefficient, according to Hill's theory (*l*), $-0''\cdot226$ (according to Hansen $-0''\cdot240$) ; hence the part of the observed coefficient due to the figure of the Earth is $-8''\cdot156$. We must therefore multiply the numbers $6''\cdot86$, $-6''\cdot42$, obtained with Hill's value of λ, by $8\cdot156\div8\cdot726$ to reduce them to the value deduced from Hansen's observed coefficient in latitude. They respectively become $+6''\cdot41$, $-6''\cdot00$.

Faye (*m*), however, gives $-8''\cdot59$ for the observed coefficient in latitude, so that the part due to the Earth's action is $-8''\cdot36$. The corresponding motions are $+6''\cdot57$, $-6''\cdot15$. Helmert (*m*) has a still different value.

The coefficient λ, according to the theories of the figure of the Earth, is approximately equal to $k\,(e_1 - \frac{1}{2}\phi)$, where k is a constant determinable with all the accuracy required here, ϕ is the ratio of centrifugal force to gravity at the equator and is equal to ·003468, and e_1 is the ellipticity of the Earth's surface.* As is well known, this equation is obtained on certain suppositions concerning the constitution and distribution of the Earth's mass. Hill (k) finds λ from a collection of pendulum observations, and deduces $1/e_1 = 286{\cdot}7$. Helmert (m) has deduced $1/e_1 = 298{\cdot}3 \pm 1{\cdot}0$, also from pendulum observations. Clarke (m), from geodetical measurements, obtains $1/e_1 = 293{\cdot}5 \pm 1{\cdot}0$. The precessional constant used by Newcomb (b) is connected by the theory of the figure of the Earth with the value 297·3. As the value of the ellipticity deduced from Hill is greater than most of the other determinations, it seems worth while to give a list of the principal determinations, together with the corresponding motions of the perigee and node :

Author.	$\frac{1}{e_1}$	Annual Motions. Perigee.	Node.
Hill, pendulum obs.	286·7	+ 6″·86	− 6″·42
(α) Faye, coef. in lat.	292·9	+ 6·57	− 6·15
Clarke, geodesy	293·5	...	...
(β) Hansen, coef. in lat.	296·3	+ 6·41	− 6·00
Precessional const.	297·3	...	...
Helmert, Moon pert.	297·8	...	...
Helmert, pendulum obs.	298·3	+ 6·33	− 5·94

Hill's theoretical coefficient in latitude is compared with Faye's observed coefficient in order to find e_1 in (α) of the table, and Hansen's observed coefficient is similarly treated, so that Hansen's theoretical coefficient does not enter into this discussion.

According to the evidence Hill's value is too large. Faye's value puts the differences between the observed and theoretical motions of the perigee and node within the limits of errors arising from other known sources. Hansen's value is close to that obtained by Helmert, and also to that deduced from the precessional constant. I shall therefore in the final results, awaiting a new discussion of the observed value of the coefficient in latitude, give the two values (α), (β), these being practically the values round which a choice seems probable ; they also make the theory consistent as far as the adopted values

* The ellipticity is defined by $e_1 = (a-c)/a$, where a, c are the equatorial and polar axes of the Earth. The complete relation between λ, e_1 is given by Professor Darwin in the *Monthly Notices* for 1899 December. I am much indebted to Professor Darwin for assistance in the discussion of this paragraph, and to Professor Helmert for communicating a calculation of the values of ϕ, k, from the relation just mentioned, and of the latest result, 298·3 ± 1·0 for $1/e_1$ (*Berl. Sitzungsber.*, 1901, pp. 328–336).

of the constants are concerned. A change of 1 per cent. in the ellipticity causes changes of 2 per cent. in the corresponding perturbations of the Moon's motion.

8. *Other perturbations.*—Hansen (*f*) considers the figure of the Moon as responsible for the differences he finds between the observed and theoretical motions—namely, $+1''\cdot50$, $-2''\cdot56$. As will be seen, no such differences exist, and in any case modern measurements of the Moon seem to put the large values Hansen assigned for the ellipticity of the Moon entirely out of the question.* In fact, until the question of the ellipticity of the Earth is settled it is unnecessary to invoke any other sources of perturbation beyond those which can be definitely calculated. In particular the law of gravitation at the distance of the Moon appears to be exact within the limits of accuracy at present attainable.

9. *The final results.*—Gathering together the results in the foregoing sections, we obtain the following :

Annual Mean Motions, Epoch 1850·0.

		Perigee.	Node.
Solar Action	char^{c}. 1	$+148\ 524''\cdot92$	$-69\ 287''\cdot90$
	,, e^2	$-\ 519\cdot31$	$-\ 616\cdot09$
	,, γ^2	$-\ 1\ 739\cdot85$	$+\ 260\cdot59$
	,, e'^2	$+\ 156\cdot27$	$-\ 25\cdot46$
	,, α^2	$+\ 2\cdot24$	$-\ 1\cdot11$
	,, e^4	$+\ \cdot04$	$+\ \cdot07$
	,, $e^2\gamma^2$	$+\ 6\cdot72$	$-\ 1\cdot70$
	,, γ^4	$-\ 1\cdot51$	$+\ \cdot05$
	,, $e^2e'^2$	$-\ \cdot99$	$-\ \cdot57$
	,, $\gamma^2e'^2$	$-\ 1\cdot61$	$+\ \cdot08$
	,, rem^{s}.	$\cdot00 \pm \cdot04$	$\cdot00 \mp \cdot02$
Terms in No. 2		$-\ \cdot70$	$+\ \cdot20$
Terms in No. 3		$+\ \cdot01$	$-\ \cdot01$
Plan. direct		$+\ 2\cdot66$	$-\ 1\cdot42$
Plan. indirect		$-\ \cdot19$ $\} \pm \cdot06$	$+\ \cdot05$ $\} \mp \cdot03$
Fig. of earth (α)		$+\ 6\cdot57$	$-\ 6\cdot15$
Fig. of earth (β)		$+\ 6\cdot41$	$-\ 6\cdot00$
Calcul. sum (α)		$+146\ 435\cdot27 \pm \cdot10$	$-69\ 679\cdot37 \mp \cdot05$
Calcul. sum (β)		$+146\ 435\cdot11 \pm \cdot10$	$-69\ 679\cdot22 \mp \cdot05$
Observed		$+146\ 435\cdot23$	$-69\ 679\cdot45$
C−O (α)		$+\ 0\cdot04 \pm \cdot10$	$+\ 0\cdot08 \mp \cdot05$
C−O (β)		$-\ 0\cdot12 \pm \cdot10$	$+\ 0\cdot23 \mp \cdot05$

* It is to be noticed that if we add 1/12 of these differences to the result (β) in No. 9, the differences C−O (β) practically disappear. But both sets are too doubtful to build any argument on their possible existence.

Of the calculated results (α), (β) in the above table, the former is obtained by using Faye's value of the observed coefficient in latitude to determine the constant on which the terms due to the figure of the Earth depend, the latter by using Hansen's observed coefficient. It is probably true that Hansen's coefficient refers to his special axes, but the difference between his and the ecliptic system will not materially alter it. The errors $\pm$·10, $\mp$·05 are the extreme errors previously explained.

Nothing has been said concerning the probable errors of the observed values of the mean motions. In view of Newcomb's (*a*) confirmation of Hansen's (*f*) results (a change of 0″·1 in the motion of the node alone being made), it would seem that there is little doubt of their accuracy as far as Hansen's theory with the known corrections is concerned. But of late years the examination of planetary inequalities has revealed several whose periods are very close to that of the principal elliptic term. Three of them have coefficients approaching 1″ in magnitude, according to Radau (*n*), and it is not impossible that a systematic search may reveal others whose inclusion in the theory may cause slight alterations in the observed values of the mean motions of the perigee and node. The values adopted above are those of Newcomb (*a*).

10. *Bibliography.*—The references above are to the following papers :

(*a*) S. Newcomb, "Investigation of Corrections to Hansen's Tables of the Moon," *Transit of Venus Papers*, pt. 3, Washington, 1876.

(*b*) S. Newcomb, "Tables of the Sun," *Washington Astr. Papers*, vol. vi. pt. 3, 1895.

(*c*) E. W. Brown, "Theory of the Motion of the Moon," pt. 1. *Mem. R.A.S.* vol. liii. 1897.

(*d*) E. W. Brown, "On certain Properties of the Mean Motions...," *Proc. Lond. Math. Soc.* vol. xxviii. 1896.

(*e*) E. W. Brown, "On the Mean Motions of the Lunar Perigee and Node," *M.N.R.A.S.* vol. lvii. 1897. Note on the same, *ib.*

(*f*) P. A. Hansen, "Darlegung...," pt. 1, *Sächs. Abh.* vol. vi. 1862.

(*h*) S. Newcomb, "Action of the Planets on the Moon," *Washington Astr. Papers*, vol. v. pt. 3, 1894.

(*i*) S. Newcomb, "Periodic Perturbations . . . of the Four Inner Planets . . ." *Washington Astr. Papers*, vol. iii. pt. 5, 1891.

(*k*) G. W. Hill, "Inequalities . . . produced by the Figure of the Earth," *Washington Astr. Papers*, vol. iii. pt. 2, 1891.

(*l*) G. W. Hill, "On the Lunar Inequalities produced by the Motion of the Ecliptic," *Annals of Mathematics*, vol. i. 1884.

(*m*) These references are extracted from F. Tisserand, *Méc. Cél.* vol. ii. chap. 21, and W. Harkness, "Solar Parallax and Related Constants," *Washington Observations for* 1885, app. 3.

(*n*) R. Radau, "Recherches concernant les Inégalités plané-

taires du mouvement de la Lune," *Par. Obs. Ann.* (*Mém.*) vol. xxi. 1892.

The work of Harkness (*m*) contains a collection of all the constants of astronomy and a full bibliography.

Haverford College: 1904 *January* 4.

The Parallactic Inequality and the Solar Parallax.

By Ernest W. Brown, Sc.D., F.R.S.

Of the principal solar terms in the Moon's motion the parallactic inequality is the only one whose coefficient can be determined correctly within $0''\cdot1$ when characteristics of order higher than the third are omitted. Like all the terms whose coefficients contain a/a' as a factor, the convergence of its coefficient along powers of m is slow, and consequently its value deduced from Delaunay's literal expression, even when allowance is made for higher powers of m, is not certain within half a second. The transformation of my theory to polar coordinates up to and inclusive of the terms whose characteristics are of order 3 was completed some time ago (*Monthly Notices*, 1899 December). As the coefficient of the parallactic inequality can be found, with a given set of constants, correctly to $\frac{1}{5000}$ of the whole from these terms it may perhaps be useful to give the result.

The following table gives the values of the various portions of the coefficient obtained by myself and Delaunay. It is expressed exactly in the form and with the constants of Delaunay, who uses a solar parallax of $8''\cdot75$. The column headed l.p. *m.* gives the value of the last power of m calculated by Delaunay. The result has not been multiplied by $(E-M)/(E+M)$, the factor necessary to obtain the final value.

Coefficient of Parallactic Inequality.

Characteristic.	Brown.	Delaunay.	l.p. *m.*
$a/a'=\alpha$	$-128''\cdot069$	$-127''\cdot621$	$-0''\cdot381$
$e^2\alpha$	$-\ 2\cdot485$	$-\ 2\cdot378$	$-\ \cdot119$
$e'^2\alpha$	$-\ \cdot022$	$-\ \cdot024$	$+\ \cdot001$
$\gamma^2\alpha$	$+\ 3\cdot105$	$+\ 2\cdot772$	$+\ \cdot354$
α^3	$-\ \cdot001$	$\cdot000$	$-\ \cdot0004$
Higher	...	$+\ \cdot009$	$+\ \cdot009$
Sum	$-127\cdot472$	$-127\cdot242$	...

My result is thus $-127''\cdot47$ + higher terms. The latter, judging from Delaunay's calculation and the experience gained as to the

general rate of convergence of particular classes of terms, will give $+0''\cdot02\pm0''\cdot02$. Thus the coefficient is $-127''\cdot45\pm0''\cdot02$. Multiplying this by $(E-M)/(E+M) = 80\cdot15/82\cdot15$, and reducing to the parallax $8''\cdot790$, I find

$$\text{parallactic inequality} = -124''\cdot92\,\frac{\text{solar parallax}}{8''\cdot790}\sin D$$

with a possible error of $0''\cdot02$.

In the *Monthly Notices* for 1903 December Mr. Cowell finds $-124''\cdot75$ for the observed value of this coefficient. Comparing we obtain

$$\text{solar parallax} = 8''\cdot778.$$

Haverford College: 1904 *April* 1.

Some further Analyses of the Moon's Errors of Longitude, 1847–1901. By P. H. Cowell.

The columns 2–7 of the annexed table gives the "apparent" coefficient of $\sin 2\theta$, $\cos 2\theta$, $\sin(4D-2g)$, $\cos(4D-2g)$, $\sin D$ and the apparent error of semi-diameter as deduced from each of the forty-eight periods of analysis, consisting of 400 lunar days each, into which the period 1847–1901 has been divided.

Last month I explained how an apparent coefficient of $\sin\phi$ might in reality be due to a term $\sin(\phi\pm D)$. Periodicity in the apparent coefficient of $\sin\phi$ indicates a term different slightly in argument from ϕ or from $\phi\pm D$.

The angle θ is an auxiliary angle whose movement is $13^\circ\cdot5$ in a lunar day. θ therefore recurs in eighty days and 2θ in forty days; θ differs very slightly in movement from g, the mean anomaly, for the movement of g in 400 lunar days $= 13\cdot5\times400 + 9^\circ\cdot1681$. $4D-2g$ is double the evection.

The first four coefficients tabulated appear to be nearly, if not entirely, accidental. I cannot find any marked periodicity in any one of the four series. The conclusion from this is that the tables of Hansen contain either no error (with coefficient exceeding $0''\cdot3$ say) and argument $2g$, $4D-2g$, $2g\pm D$, $4D-2g\pm D$ or angles differing from any of the above by an argument of long period, such as $\omega-\omega'$ or 2ω, or that if an error in one term does exist, its apparent effect is cancelled (within a limit of about $0''\cdot3$) by another error in an allied term or errors in both the allied terms—an improbable supposition. Hansen's tables are therefore probably correct as regards terms of the above form. Subsequently I intend to give analyses of the allied terms $g+g'$ and $g-3g'$.

Apart from accidental error, therefore, I infer that all these

In Tenths of a Second of Arc.

Period.	Apparent Coefficients of					Apparent Corr. to Semi-diameter.	Resolved Coefficient of sin D.	Resolved Corr. to Semi-diameter.
	sin 2θ.	cos 2θ.	sin (4D−2g).	(cos 4D−2g).	sin D.			
86	−2	+3	+2	−2	+2	+1	+5	−2
87	−4	+6	−7	−9	−11	−8	−7	−3
88	0	+2	0	+2	−1	−2	+8	−8
89	+4	+3	0	−4	0	0	0	0
90	−6	−2	−2	+3	−5	−4	−1	−4
91	+5	−1	+3	+3	+5	+3	+8	−2
92	+4	−5	−4	−1	−8	−5	−10	+2
93	−2	−1	0	+1	0	−2	+14	−12
94	+3	−1	+5	−2	−5	−3	−8	+2
95	+8	−8	0	0	−9	−6	−9	0
96	−3	−1	−1	−2	−12	−8	−12	0
97	−9	+2	+1	+2	−13	−11	+3	−14
98	+3	+4	+5	+3	−9	−6	−9	0
99	−3	−3	+3	0	−1	0	−6	+4
100	−1	+5	−2	−1	+4	+3	+2	+2
101	+8	−4	+2	0	−1	0	−6	+4
102	−5	+1	−2	−3	+4	+3	+2	+2
103	−6	0	−1	0	+4	+3	+2	+2
104	+1	+7	0	−2	−6	−3	−14	+6
105	+9	−1	+1	−3	+1	+1	−1	+2
106	−2	−3	−2	0	+3	+1	+10	−6
107	0	+4	0	+1	+2	+1	+5	−2
108	−5	−6	+2	−4	+6	+4	+7	0
109	+1	+4	0	0	+13	+8	+18	−4
110	−3	0	−7	+3	+2	+3	−9	+9
111	+5	+4	−2	−4	−3	−2	−3	0
112	−1	−4	0	+8	−1	0	−6	+4
113	−7	+1	0	+2	+2	+1	+5	−2
114	−1	+5	−1	−3	+3	+2	+3	0
115	−2	−3	+6	+3	+3	+3	−4	+5
116	0	+5	0	−1	+3	+3	−4	+5
117	+5	+1	−6	+1	+4	+4	−5	+8
118	+1	−6	+3	+1	+4	+3	+2	+2
119	−1	−2	−6	0	+2	+1	+5	−2
120	−5	+4	+4	+3	0	0	0	0
121	−4	+1	+2	−1	+5	+3	+8	−2
122	−5	+1	+2	−1	+12	+8	+12	0
123	−5	+5	+1	0	+5	+3	+8	−2
124	+5	+4	−2	−4	+5	+3	+8	−2
125	−2	+1	+1	+1	+1	+1	−1	+2
126	−1	+4	+3	−4	−6	−4	−7	0
127	+5	+2	+4	−1	+2	+2	−2	+4
128	+2	−4	−1	+3	−1	0	−6	+4
129	−5	−3	+1	+1	−4	−4	+5	−8
130	0	+3	+2	+2	−5	−3	−8	+2
131	+2	−2	0	−3	−6	−5	+1	−6
132	−6	+1	0	0	−2	−1	−5	+2
133	+7	+4	−2	−6	−2	−1	−5	+2

quantities should be zero. The probable error of the apparent coefficient of any periodic term can therefore be deduced. Adding the four columns without regard to sign and dividing by 48, and then multiplying by 0·85 to reduce "Mean of the Errors" to "Probable Error," I obtain from the apparent coefficient of

$\sin 2\theta$,	174	÷ 48 × 0·85	=	±0″·31
$\cos 2\theta$,	147	,, ,,	=	±0″·26
$\sin(4D-2g)$,	101	,, ,,	=	±0″·18
$\cos(4D-2g)$,	104	,, ,,	=	±0″·18

and from the mean of these four separate determinations the probable error of any entry in the first four columns of the table seems to be about ±0″·23.

With this last number as the basis, the probable error of a single "tabular — observed" error of the Moon appears to be about $\pm 0''\cdot 23 \times \sqrt{\frac{1\cdot 5647}{48}} = \pm 1''\cdot 7$. In my December paper I gave 1″·5 as an estimate of this quantity.

Again, the probable error of a coefficient deduced from all forty-eight periods is

$$\pm 0''\cdot 23 \div \sqrt{48} = \pm 0''\cdot 03$$

When the Airy period 1750–1851 is fully worked up there will be plenty of material for checking this estimate. At present I can only refer to the values of the principal elliptic coefficient and the coefficient of the Jovian evection in my paper of last month, from which it will be seen that 0″·03 is a fair estimate of the probable error.

As regards the tables of the Moon the results of these two analyses are of a negative character only, indicating the absence and not the presence of periodic errors. It is interesting to note, however, that had Airy analysed for the argument $4D-2g$ he would have found a periodic error of 4″ in his tabular places, and this would probably have led to a more thorough analysis.

Coming now to the last four columns of the table the sixth column exhibits the apparent coefficient of sin D, that is to say, the quantity obtained by multiplying each error by sin D and dividing by half the number of observations. It is evident that an error of semi-diameter, or a term sin 2D, will each contribute to the "apparent coefficient of sin D." The seventh column exhibits the apparent error of semi-diameter, that is to say, the quantity obtained by reversing the errors of the second limb, adding on the errors of the first limb and dividing by the whole number of observations. It is evident that terms in sin D and sin 2D will contribute to the apparent error of semi-diameter.

I copy here the left-hand sides of the normal equations

obtained in my December paper, where I attempted to separate the semi-diameter, parallactic inequality, and variation.

$$+5647\mu+3788\delta_1-2327\delta_2 \ (=5647 \text{ apparent } \mu)$$
$$+3788\mu+3074\delta_1-1366\delta_2 \ (=3074 \text{ apparent } \delta_1)$$
$$-2327\mu-1366\delta_1+2645\delta_2 \ (=2645 \text{ apparent } \delta_2)$$

On analysing the apparent coefficient of sin D and the apparent correction to semi-diameter for a period $\omega-\omega'$ I obtain

(in the coefficient of sin D)	$+0''\cdot29\cos(\omega-\omega')$
(in the correction to semi-diameter)	$+0''\cdot19\cos(\omega-\omega')$

The first of the normal equations shows that a real inequality $+0''\cdot29\cos(\omega-\omega')$ in the coefficient of sin D will produce an apparent inequality $+0''\cdot29\times3788\div5647 = +0''\cdot19\cos(\omega-\omega')$ in the apparent correction to semi-diameter. Hence the conclusion is that there is no periodic inequality in the true correction to semi-diameter, and that the correction $-0''\cdot29\cos(\omega-\omega')\sin D$ is really required by the tabular places.

No attempt has yet been made to analyse the observations for a term cos D. On account of the grouping of the observations round full Moon the analysis will clearly be subject to an increased accidental error. Hence it is impossible to say at present in what proportions the term $-0''\cdot29\cos(\omega-\omega')\sin D$ must be distributed between arguments $g-g'$ and $g-g'+2\omega-2\omega'$.

The following are the coefficients of sin $(g-g')$ and sin $(g-g'+2\omega-2\omega')$ according to various authorities :

	$\sin(g-g')$.	$\sin(g-g'+2\omega-2\omega')$.
Hansen's tables	$+148''\cdot03$	$-28''\cdot60$
„ theory	$+148''\cdot02$	$-28''\cdot56$
Delaunay (1)	$+147''\cdot46$	$-29''\cdot50$
„ (2)	$+148''\cdot43$	$-28''\cdot32$

The last three lines are taken from Newcomb's paper (*Astron. Papers Amer. Eph.* vol. i.). Delaunay (1) and Delaunay (2) are defined in that paper. The difference is the result of estimating for the higher powers not calculated by Delaunay. It is well known that in these terms (that is, terms with characteristic ee') the convergence of Delaunay's series is extremely slow. The present paper shows that in respect of these terms Hansen's theory is considerably nearer the observations than either Delaunay (1) or Delaunay (2).

Lastly I have solved for "true" (in contradistinction to

"apparent") corrections to the coefficient of sin D and the semi-diameter by the formulæ

$$\mu = 5\cdot77 \text{ apparent } \mu - 3\cdot87 \text{ apparent } \delta_1$$
$$\delta_1 = 5\cdot77 \text{ apparent } \delta_1 - 7\cdot11 \text{ apparent } \mu$$

and the results are given in the last two columns.

From the normal equations above quoted it will be seen that the left-hand sides are strictly $\mu - 0\cdot7\delta_2$ and $\delta_1 + 0\cdot3\delta_2$; but as $\delta_2 = +0''\cdot03$ the difference is unimportant.

Since the December paper was written I have corrected the observations individually for an error of semi-diameter $0''\cdot4$. Judging from that paper, therefore, the solution to be expected is

$$\mu = -0''\cdot054 \qquad \delta_1 = -0''\cdot120$$

From the means of the last two columns in the present paper I get

$$\delta_1 + 0''\cdot01 = -0''\cdot021$$
$$\mu - 0''\cdot02 = -0''\cdot015$$

differing very slightly from the previous values.

Adding the last four columns without regard to sign I obtain for the probable error of the results of the separate periods

for δ_1 (apparent)	...	...	$211 \div 48 \times 0\cdot85 =$	$\pm 0''\cdot38$
„ μ „	...	...	$148 \div 48 \times 0\cdot85 =$	$\pm 0''\cdot26$
„ δ_1	...	...	$292 \div 48 \times 0\cdot85 =$	$\pm 0''\cdot52$
„ μ	...	...	$157 \div 48 \times 0\cdot85 =$	$\pm 0''\cdot28$

From the method of formation of the first of these quantities ($\pm 0''\cdot38$) there is no reason why it should be larger than the mean ($\pm 0''\cdot23$) of the four similarly formed quantities previously obtained. That it is larger is perhaps evidence that the errors are not entirely accidental with reference to the argument D, or that there are time changes from one period of analysis to another, such as might be introduced by personality in semi-diameter and a change of observers. The second quantity is also large, considering that it has double weight. $0\cdot26 \times \sqrt{2} = 0\cdot36$, and this quantity is therefore comparable with the preceding. The third quantity is determined with a weight diminished in the ratio 1030 : 5647 (see December *Monthly Notices*, footnote); $\pm 0''\cdot52$ is in close agreement therefore with $\pm 0''\cdot23$ obtained previously found. The fourth quantity is determined with a weight diminished in the ratio 1562 : 5647; $\pm 0''\cdot28$ is therefore comparable with a probable error $\pm 0''\cdot15$ in the coefficients of periodic inequalities. This additional evidence is therefore against the hypothesis that there are time-changes, for it appears

that the values of μ are more accordant than might have been expected, having regard to accidental error only.

On analysing the last column but one for argument Ω I find

(tabular *minus* observed) $(+0''.40 \sin \Omega - 0''.12 \cos \Omega) \sin D$

The former term is far too large to be accidental; but as $\sin \Omega \sin D$ splits into cosines, and not sines, it is probable that Ω is not the actual argument of the periodicity, but only a near approximation to it. The point is accordingly held over till the Airy period is reduced. As far as I can foresee, this reduction should be complete in about two months.

There is a point to which attention should be called, although I am at present unable to offer an explanation. The *apparent* coefficient of sin D and the *apparent* correction to semi-diameter are naturally of the same sign as a rule; in fact, they are never of unlike sign. One test of the validity of the solution into true corrections is that there shall be no periodicity in the correction to semi-diameter, but that all the periodicity should belong to the correction to parallactic inequality. The results stand this test. There is, however, another test, viz. that the signs should be independent. Now in the forty-eight periods we have thirty-two cases of unlike sign, six of like sign, and ten cases where one at least of the quantities is zero. This test is therefore not satisfied. As a similarity of sign has been replaced by, on the whole, a dissimilarity, I conclude that the process of separation has been overdone.

Note on Elliptic Motion. By Asaph Hall.

The difference of the ratio of the radius vector and the mean distance from unity and the difference of the true and mean anomalies, or the equation of centre, are two important quantities in the theory of this motion. If these quantities are wanted in explicit terms of the eccentricity and the mean anomaly, perhaps the easiest method is to apply the theorem of Lagrange and to use the equation

$$dv = \sqrt{1-e^2} \cdot \frac{a^2}{r^2} \cdot dnt.$$

The ratio $\frac{a^2}{r^2}$ can be found by the above theorem, and a substitution and integration will give the equation of centre, or $v-nt$. Bessel objected to this method as not being the simple and natural solution of the question.

In his *Mécanique* Poisson has indicated a method of obtaining these quantities by definite integrals, and this is the method

followed by Bessel. This problem is old, and the coefficients are accurately known. The aim of the writers is to find general expressions for the coefficients, which are complicated for the equation of centre, and I wish to point out that the coefficients can be found directly and easily from the series. Assume with Poisson,

$$r = A_0 + A_1 \cos nt + A_2 \cos 2nt + \ldots + A_i \cos int + \ldots$$
$$v - nt = B_1 \sin nt + B_2 \sin 2nt + \ldots + B_i \sin int + \ldots$$

If i and i' are positive whole numbers, and different, the integrals of the differentials $\cos int \cos i'ntd.nt$ and $\sin int \sin i'ntd.nt$ are zero between the limits v and π. If $i = i'$, the value is $\frac{\pi}{2}$. Hence

$$A_i = \frac{2}{\pi}\int_0^\pi r \cos int \,.\, dnt$$
$$B_i = \frac{2}{\pi}\int_0^\pi (v - nt) \sin int \,.\, dnt$$

For $i = 0$, we have an exception, and

$$A_0 = \frac{1}{\pi}\int_0^\pi r \,.\, dnt = \frac{a}{\pi}\int_0^\pi (1 - e\cos u)^2 \,.\, du = a\left(1 + \frac{e^2}{2}\right)$$

where u is the eccentric anomaly. Since the anomalies have the values 0 and π at the same time, by means of the known values of the differentials of v and nt we change the variable to u, integrate by parts, and omit the second part of B_i, which is a complete differential, and its integral is zero at the limits. Thus we have

$$A_i = -\frac{2ae}{i\pi}\cdot\int_0^\pi \sin(iu - ie\sin u)\sin u du$$
$$B_i = \frac{2\sqrt{1-e^2}}{i\pi}\cdot\int_0^\pi \frac{\cos(iu - ie\sin u)\,du}{1 - e\cos u}$$

By expansion

$$\sin(iu - ie\sin u)\sin u du$$
$$= +\sin iu\left(\sin u - \frac{i^2e^2\sin^3 u}{\lfloor 2} + \frac{i^4e^4\sin^5 u}{\lfloor 4} - \frac{i^6e^6\sin^7 u}{\lfloor 6} + \ldots\right)du$$
$$-\cos iu\left(ie\sin^2 u - \frac{i^3e^3\sin^4 u}{\lfloor 3} + \frac{i^5e^5\sin^6 u}{\lfloor 5} - \frac{i^7e^7\sin^8 u}{\lfloor 7} + \ldots\right)du$$

In order to integrate and find the coefficients A_i, we need only a table for converting powers of $\sin u$ into sines and cosines of the multiple arcs. Then by making i equal to 1, 2, 3, 4 . . . successively we pick out the term that gives the value $\frac{\pi}{2}$, multi-

plied by a power of e, and a numerical coefficient. The rest of the terms vanish. When i is even, the first line is zero ; and when i is odd, the second line is zero. As a trial I have computed in this way the twenty-five terms of the A_i, the coefficients including the ninth power of e. Comparing these, without revision, with Le Verrier's I made only three errors. The work is very simple.

Expanding the differential part of B_i, we have

$$+\left(1-\frac{i^2e^2\sin^2 u}{\lfloor 2}+\frac{i^4e^4\sin^4 u}{\lfloor 4}-\frac{i^6e^6\sin^6 u}{\lfloor 6}+\ldots\right)\cos iu\,.\,du$$

$$+\left(ie\sin u-\frac{i^3e^3\sin^3 u}{\lfloor 3}+\frac{i^5e^5\sin^5 u}{\lfloor 5}-\ldots\right)\sin iu\,.\,du$$

$$+\left(e-\frac{i^2e^3\sin^2 u}{\lfloor 2}+\frac{i^4e^5\sin^4 u}{\lfloor 4}-\frac{i^6e^7\sin^6 u}{\lfloor 6}+\ldots\right)\cos iu\cos u\,.\,du$$

$$+\left(ie^2\sin u-\frac{i^3e^4\sin^3 u}{\lfloor 3}+\frac{i^5e^6\sin^5 u}{\lfloor 5}-\ldots\right)\sin iu\cos u\,.\,du$$

&c. &c. &c.

The first two lines can be integrated as before. For the third line an integration by parts gives the partial value

$$B'_i=\frac{2}{\pi}\cdot\frac{\sqrt{1-e^2}}{n+1}\cdot\int_0^\pi \sin^{n+1}u\sin iu\,.\,du$$

and the fourth line

$$B''_i=-\frac{2}{\pi}\cdot\frac{\sqrt{1-e^2}}{n+1}\cdot\int_0^\pi \sin^{n+1}u\cos iu\,.\,du$$

The lines that follow in the expansion will return to the above forms. n is any exponent of $\sin u$, and for any value of n we make i, 1, 2, 3, 4 . . . &c. Here the work is longer, and we have to multiply by the expanded form of $(1-e^2)^{\frac{1}{2}}$.

Hansen has given some elegant forms for the equation of centre. He puts $e=\sin\phi$, and expands in powers of $\beta=\tan\frac{1}{2}\phi$. His β is the same as the λ of Laplace. The objection of Bessel to the method of Lagrange does not appear to be well founded. It should be a function of the higher analysis to furnish better methods for practical work.

Goshen, Conn.:
1904 *March* 21.

The Rousdon Variable Star Observations.

By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The observations of long-period variables made during the years 1886–1900 at the Rousdon Observatory, Lyme Regis, Devon, under the direction of the late Sir C. E. Peek, Bart., were put into my hands for discussion and publication after Sir C. E. Peek's death, and will appear as Vol. LV. of the *Memoirs*. A brief account of the discussion may be convenient for other observers.

2. Comparisons were made whenever possible between the variable and *five* stars within a field approximately 1° square. It was of course impossible always to find five stars in this area within a few tenths of a magnitude of the variable; and the differences estimated often exceed a magnitude, sometimes two or three. The accidental errors of such comparisons are naturally large; but there are, nevertheless, distinct advantages in the method which appear on discussion. As regards systematic error it appears that a certain quantity (about 0·2 mag.) must be *numerically subtracted* from all estimated differences greater than about ±0·4 mag. But the value of this quantity changes sensibly with the brightness of the stars compared, and also at two epochs during the work. Still it is not difficult to correct with tolerable accuracy for this arbitrary scale.

3. There is some evidence of systematic error depending on position-angle, but it is conflicting; and, after several vain attempts to establish laws for it from the material available, the quest was abandoned.

4. There seems to be no doubt that the magnitudes for the comparison stars determined at Harvard will not in all cases suit the Rousdon observations. Mr. Grover sees some of them systematically brighter or fainter, doubtless owing to peculiarities of colour. Some trouble was taken to deduce individual magnitudes which would suit his observations, while yet conforming generally to the Harvard scale. But the consequent corrections have been exhibited in a separate column (as also those for the scale value) so that they can be omitted if this is thought desirable.

5. A comparison with the Harvard observations (*Harvard Annals*, vol. xxxvii., Part I.) is given. Two important deductions are made from this comparison.

(*a*) *Systematic Errors.*—The stars observed were divided into two classes, A and B, according to their "redness" as given in Chandler's Third Catalogue. Six stars in Class A (viz. *R*, *S*, and *T Cassiopeiæ*, *S* and *T Cephei*, and *R Aurigæ*) have a mean redness 7·1; and five stars in Class B (viz. *S* and *T Ursæ Majoris*, *S Boötis*, *R Camelopardi*, and *R Draconis*) have a mean redness 2·4. The differences, Rousdon—Harvard, were

arranged according to the mean Rousdon magnitude, and gave the following mean results:

Rousdon Mag.	Redness.	6·5	7·5	8·5	9·5	10·5	11·5	12·5
Class A ...	7·1	+·6	+·3	−·1	−·3	−·6	−·7	−1·1
Class B ...	2·4	...	0	0	0	−·1	0	+·2

There is thus a progressive difference *for red stars* amounting to 0·3 per magnitude! This personal equation far exceeds any other systematic error discussed in the *Memoir*, and seems to demand the most careful investigation if series of observations by different observers are to be combined. It throws some light on the special results for comparison stars mentioned in the last paragraph. It does not affect discussions of observations by the same observer unless he is found to change his habit; but it seems quite possible that there may be such changes of habit, and a series of comparisons between observers at different times would give valuable information.

T Cephei. Assumed

Residuals for each

Group. Julian Day.	1 0	2 32	3 64	4 96	5 128
1871	...	...	...	...	...
2255	− ·3	− ·2	− ·5	+ ·2	+ ·2
2639	− ·3	− ·6	− ·5	+ ·3	+ ·3
3023	+ ·4	− ·3	+ ·1	− ·1	+ ·5
3407	+ ·4	+ ·7	+ ·5	− ·1	+ ·5
3791	+ ·6	+ ·2	+ ·7	+ ·6	+ ·2
4175	+ ·5	+ 1·0	+ ·1	+ ·1	(− ·2)
4559	− ·5	− ·1	− ·3	− ·7	− ·8
4943	− ·2	+ ·6	·0	...	...
Mean	+0·08	+ 0·16	+0·01	+0·04	+0·10
Mag. from Curve ...	7·50	6·80	6·38	6·83	7·56
2255+(2)	− ·2	− 1·1	− ·9	+ ·4	+1·0
3791+(2)	+ ·6	+ 1·1	+ ·5	·0	− ·8
Difference	+ ·8	+ 2·2	+1·4	− ·4	−1·8
Factor 5·09	+4·1	+11·2	+7·1	−2·0	−9·2
$p-f$	+0·6	+ 1·2	+0·1	−1·3	−1·4
Corr. Period... ...	+6·8	+ 9·3	....	+1·5	+6·6

(*b*) *Accidental Errors.*—There are occasions when an observer records a sensible deviation from a mean curve, amounting to more than half a magnitude. It is very important to know whether these deviations are real or accidental. Observers themselves sometimes feel convinced of their reality. It is interesting to find that, in the case of *T Cassiopeiæ*, at any rate, which is discussed in detail in the *Memoir*, when a sensible deviation of this kind is observed at Rousdon it is not confirmed at Harvard, and *vice versâ*. Systematic difference was allowed for before making the comparison. From this and other evidence it would seem that the variation of light in these stars is possibly a strictly periodic phenomenon, though several waves may be superposed.

6. Besides tables giving the observations for each night separately, compendious tables have been formed summarising the results. The following table for *T Cephei* will sufficiently illustrate the method adopted :

period, 384 days.

Group of 32 Days.

6 160	7 192	8 224	9 256	10 288	11 320	12 352
...	+ 1·1	+ ·1	·1	− ·3	− ·5	− ·1
+ ·3	+ ·5	+ ·2	+ ·6	− ·1	·0	+ ·2
·0	(− ·1)	− ·2	− ·5	− ·6	·0	− 4
·0	− ·2	+ ·3	+ ·7	− ·2	− ·1	+ ·3
+ ·1	− ·4	+ ·4	− ·1	− ·1	+ ·1	+ ·5
+ ·1	− ·2	(·0)	+ ·2	+ ·4	+ ·4	+ ·6
− ·5	− ·8	− 1·1	− ·3	− ·1	− ·1	− ·6
− ·6	− 1·0	− ·7	− ·4	− ·1	+ ·2	− ·7
...	...	...	...	...	...	...
−0·09	− 0·14	− 0·12	+0·03	−0·14	0·00	−0·03
8·32	9·07	9·62	9·53	8·93	8·10	7·60
+ ·3	+ ·2	+ ·3	+ ·8	− ·9	− ·1	+ ·1
−1·0	− 2·0	− 1·8	− ·5	+ ·2	+ ·5	− ·7
−1·3	− 2·2	− 2·1	−1·3	+1·1	+ ·6	− ·8
−6·6	−11·2	−10·7	−6·6	+6·0	+3·1	−4·1
−1·3	− 1·2	− 0·6	+0·7	+1·5	+1·2	+0·6
+5·1	+ 9·3	+17·8	−9·4	+4·0	+2·6	−6·8

7. Starting from an arbitrary date with an assumed period the observations were divided into groups of one-twelfth period and a mean light curve formed as shown in the line "Mag. from Curve." The individual observations were compared with this curve and the residuals for each group combined. The mean date to which any residual in the table corresponds is formed by adding the numbers at the top of the column and the end of the row and the omitted constant 2410000 to get the Julian date. Thus the first residual +1·1 corresponds to Julian date 2410000 +1871+192 = 2412063. From the means of the columns it is seen that the assumed light curve is not quite correct, but the corrections indicated, or any portion of them, can readily be applied.

8. In the lower part of the table it is shown how a correction to the assumed period may readily be deduced. Sums of residuals for the period 2255 and two following are denoted by 2255+(2). Comparing these with a later series, the differences are negative from maximum to minimum, and positive from minimum to maximum; indicating that the assumed period is too short. Applying the "factor 5·09" and the divisor "$p-f$" we get a correction to the period from each column, as fully explained in the memoir. The weighted mean of these corrections for *T Cephei* is +4·9 days, so that the true period deduced from the Rousdon observations is 388·9 days.

9. In this way an accurate value for the period has been deduced for each star from the Rousdon observations alone; and compared with that given in Chandler's Third Catalogue (*A. J.* No. 379). His revised "Elements" (*A. J.* No. 553) have since been received and compared with the Rousdon determinations in a postscript. The effect of the revision is so marked that the following summary of it may be repeated here. For the majority of the stars the period is variable according to a definite law, that for *S Cassiopeiæ*, for instance, being denoted by

$$610{\cdot}5\ \mathrm{E}+37\ \sin\ (15^\circ\ \mathrm{E}+59^\circ)$$

the mean period of 610·5 days being liable to regular variations of thirty-seven days each way. For comparison with Rousdon, the periods have been obtained from the formulæ by substituting the values of E appropriate to the Rousdon observations.

No.	Star's Name.	Chandler's Period. Old. d	Chandler's Period. New. d	Rousdon Period. d	R—Old. d	R—New. d
107	T Cassiopeiæ	445·0	443·5	444·4	− 0·6	+ 0·9
432	S Cassiopeiæ	606·2	619·4	630·7	+24·5	+11·3
1855	R Aurigæ	460·2	455·0	454·2	− 6·0	− 0·8
2100	U Orionis	375	375	374·4	− 0·6	− 0·6
2478	R Lyncis	380·0	377·4	377·8	− 2·2	+ 0·4
3825	R Ursæ Majoris	304·2	302·4	300·4	− 3·8	− 2·0

No.	Star's Name.	Chandler's Period. Old. d	New. d	Rousdon Period. d	R—Old. d	R—New. d
4511	T Ursæ Majoris	254·8	254·8	255·5	+ 0·7	+ 0·7
4557	S Ursæ Majoris	223·0	224·4	224·5	+ 1·5	+ 0·1
5157	S Boötis	259·8	266·5	265·9	+ 6·1	− 0·6
5190	R Camelopardi	269·5	273·9	273·3	+ 3·8	− 0·6
5504	S Coronæ	360·8	362·6	363·3	+ 2·5	+ 0·7
5955	R Draconis	245·6	245·6	245·8	+ 0·2	+ 0·2
6044	S Herculis	308·1	305·7	309·0	+ 0·9	+ 3·3
6449	T Draconis	(569)	426	428·3	...	+ 2·3
7045	R Cygni	425·7	426·6	427·0	+ 1·3	+ 0·4
7120	χ Cygni	406·2	406·2	407·0	+ 0·8	+ 0·8
7220	S Cygni	320·8	324·1	327·0	+ 6·2	+ 2·9
7609	T Cephei	387	387	388·9	+ 1·9	+ 1·9
7779	S Cephei	484	487·9	489·8	+ 5·8	+ 1·9
8600	R Cassiopeiæ	424·7	436·5	438·7	+ 14·0	+ 2·2

10. It will be seen that the new formulæ suit the Rousdon results much better than the old in every case but that of *S Herculis*. For *S Cassiopeiæ* the period is still capable of improvement; and perhaps also for *R Ursæ Majoris* and *R Cassiopeiæ*. But the general agreement is such as to warrant considerable confidence in Chandler's formulæ.

11. He gives no definite periods for *S Persei* or *R Ursæ Minoris*. For *S Persei* a main period of 840 days, and a subsidiary one of 1120 days are indicated by the Rousdon observations; though there are also traces of a much longer period (3360 days?). For *R Ursæ Minoris* the Rousdon observations indicate 320 days.

12. Some attempt is made in the Memoir to consider the possible origin of Chandler's periodic terms. The various light curves were analysed harmonically, and they seemed to fall into a series, the coefficients of the separate harmonics being related to one another. Thus dividing the stars into five groups, according to the value of A in the formula

$$A \sin \theta + B \cos \theta + C \sin 2\theta + D \cos 2\theta + E \sin 3\theta + F \cos 3\theta$$

where θ is reckoned from maximum, and the whole range of variation is represented by 6, we get the following values for the coefficients:

Group.	A	B	C	D	E	F
I.	+ ·83	−2·50	−·49	−·14	+·16	−·19
II.	+ ·40	−2·78	−·22	+·27	−·09	−·20
III.	− ·05	−2·86	+·01	+·04	−·02	−·13
IV.	− ·77	−2·70	+·30	−·23	−·03	−·04
V.	−1·19	−2·56	+·33	−·36	+·09	−·04

13. It is only natural that B should vary with A, since without the other terms, which are small, the range would be $2(A^2+B^2)^{\frac{1}{2}}$, and we have made the range constant. But there is less reason for the obvious run in the other terms. As regards D, the apparent anomaly in the value for Group I. is removed when we put the expression into the alternative form

$$H_1 \cos(\theta+G_1)+H_2 \cos(2\theta+G_2)+H_3 \cos(3\theta+G_3)$$

when the negative sign for D is seen to be merely a consequence of the continuous change in G_2. The following approximate formulæ are merely illustrative, to show roughly the dependence of all these quantities on a single variable, and have obviously no physical significance :

$$H_2 = 0{\cdot}5\, A^{\frac{3}{4}} \qquad H_3 = -0{\cdot}14 - 0{\cdot}20\, A \qquad G_2 = 130^\circ\, A$$

$$G_3 = 80^\circ\, A - 40^\circ$$

and the success with which these formulæ represent the observed quantities is shown in the following table :

				C		D				E		F	
Group.	A	H_2	G_2	Calc.	Obs.	Calc.	Obs.	H_3	G_3	Calc.	Obs.	Calc.	Obs.
I.	+ ·83	+ ·44	+ 108°	− ·42	− ·49	− ·14	− ·14	− ·31	+ 26°	+ ·13	+ ·16	− ·27	− ·19
II.	+ ·40	+ ·27	+ 52	− ·21	− ·22	+ ·17	+ ·27	− ·22	− 8	− ·03	− ·09	− ·22	− ·20
III.	− ·05	+ ·06	− 6	+ ·01	+ ·01	+ ·06	+ ·04	− ·13	− 44	− ·09	− ·02	− ·09	− ·13
IV.	− ·77	+ ·42	− 100	+ ·41	+ ·30	− ·07	− ·23	+ ·01	− 102	+ ·01	− ·03	·00	− ·04
V.	− 1·19	+ ·56	− 155	+ ·24	+ ·33	− ·51	− ·36	+ ·10	− 135	+ ·07	+ ·09	− ·07	− ·04

14. If relations of this kind can be established among the harmonic coefficients, we may be able to separate out two classes of periodic terms, one set following laws similar to those sketched above, and another with smaller amplitude and slightly different period which, by interference with the main set, cause the oscillation in epoch of maximum which Chandler represents by such expressions as

$$610{\cdot}5\, E + 37 \sin(15^\circ E + 59^\circ)$$

At any rate a promising line of investigation seems to open here. It was my intention to collect more material so as to improve the formulæ ; and I made a preliminary search for well-established light curves published by other observers in order to submit them to harmonic analysis. But I had little success in finding them—perhaps through lack of acquaintance with the literature of the subject.

15. *I should be particularly obliged if observers who have formed well-determined light curves would kindly let me have copies of them or a reference to the publication in which they are given.*

16. While writing this note it occurred to me that possibly the sun-spot curve might exhibit features either resembling those

of the variable stars or obviously differing from them. Seen as a star, the variation in total light of the Sun, owing to its variations in spottedness, is no doubt extremely small, perhaps quite insensible ; but the features of the light curves above studied seem to have little or no relation to the total range of variation, and may persist even when this has become so small as in the case of our Sun. To test the point, Wolf's sun-spot numbers for the forty-four years 1849–1892 were tabulated in four periods of eleven years,* and the sums taken as follows :

1	2	3	4	5	6	7	8	9	10	11
362	309	253	195	128	77	44	24	61	196	354

It was not difficult to draw a regular curve through these points and to read off the ordinates at twelve epochs for harmonic analysis.

17. But now came the question, Is the Sun to be regarded as (A) slightly brighter at a sunspot maximum, or (B) slightly fainter ? If spots mean loss of light (B) is the proper supposition ; but since they also mean increase of activity, it would be more natural to take (A). Both hypotheses were therefore tried, with the results given below.

Hypothesis A. Sun brighter when spotted.

18. Twelve equidistant ordinates in fractions of the greatest as unity, counting from sun-spot maximum as zero.

Max.	30°.	60°.	90°.	120°.	150°.	180°.	210°.	240°.	270°.	300°.	330°.
·00	·09	·23	·38	·54	·71	·84	·94	·99	·93	·65	·22

Analysing these by the same process as that used for the variables, we get :

	A.	B.	C.	D.	E.	F.	H_2.	G_2.	H_3.	G_3.
Observed	−1·49	−2·32	+·11	−·70	+·19	−·19	+·71	−171°	+·27	−135°
Formulæ	(−1·49)	−2·29	−·16	−·64	+·05	−·15	+·65	−194	+·16	−160

Below the "observed" values are given the values which would be deduced from the variable star formulæ given above by subtracting the value −1·49 for A. The agreement is good, the chief discrepancy being that in C, which would be removed by a comparatively slight change in the epoch G_2.

A slight change either in A or in the formula $G_2 = 130°$ A would make a satisfactory adjustment. Considering that the value of A for the Sun is much larger than in any of the series from which the formulæ were deduced, so that we are extrapolating for the Sun, the accordance may be considered satisfactory.

* For our present purpose a slight error in the period does not matter.

Hypothesis B. Sun fainter when spotted.

19. Twelve equidistant ordinates as before, but counting from sun-spot minimum as zero.

Max.	30°.	60°.	90°.	120°.	150°.	180°.	210°.	240°.	270°.	300°.	330°.
·00	·10	·42	·85	1·00	·90	·74	·60	·43	·26	·13	·05

Analysing which we get :

	A.	B.	C.	D.	E.	F.	H_2.	G_2.	H_3.	G_3.
Observed	+1·51	−2·35	−·46	−·54	−·23	+·14	+·72	+138°	−·27	+236°
Formulæ (+1·51)	−1·67	+·17	−·63	+·43	−·08	+·65	+195	−·44	+ 80	

The difficulties of reconciling the observed quantities with the formulæ are now more serious. The Hs agree as well as before ; but the discrepancy in G_2 is doubled, that in G_3 is as large as it can well be, and there is a serious discrepancy in B as calculated from the formula $B^2 = 8{\cdot}20 - 1{\cdot}8\,(A + 0{\cdot}20)^2$, deduced from the variable star results. Without a considerable modification of the formulæ, the sun-spot curve will not fit in with this end of the series at all.

20. We must not lay too much stress on the evidence, but so far as it goes it supports the conclusions :

(1) That the sun-spot curve falls into the series of variable-star curves, but outside the series presented by the Rousdon variables.

(2) That a maximum of sun-spot activity corresponds to a maximum of a variable star.

The Rotation Period of the Planet Saturn. By G. W. Hough.

In the *Monthly Notices* for January 1904, Mr. W. F. Denning suggests a wrong identification of the spots on *Saturn* as given in *Monthly Notices* for December, 1903. As so little is known regarding the time of rotation of the planet, the question is one of considerable importance. Of the spots observed on June 23–27 and July 13–29 there can be no question whatever. The latitudes agree very closely, and the spot was then conspicuous. And it was the only well-defined and conspicuous spot seen on the planet during the opposition. The spot observed by Burnham on August 19 was faint, and appeared to be out in latitude, and for these reasons its identity with the original spot may be questioned.

I think it highly probable, however, that previous to the middle of August the Barnard spot had lost its outline and ceased to be a well-defined object. The small spot observed on July 14, according to Barnard's micrometer measure, followed

thirty-nine minutes. The observation of August 19 was made when the planet was only thirty minutes from the meridian, and if the spot observed was not the Barnard spot, the other ought to have been visible on the disc. If, however, the observation of August 19 is rejected as doubtful, my results for rotation period remain substantially the same.

Since the rotation period was increasing the mean period should be slightly less than when the observation of August 19 was included. A mean period $R = 10^h\ 38^m\ 24^s$ yields the following :

Date 1903.	Days.	Obs.	O—E. m
June 23	−4	Bar	+1·4
27	0	Ho	+0·0
July 13	16	Ho	−3·0
29	32	β	+1·3

The observations are all satisfied with a mean error $\pm 1^m\cdot 4$.

According to Mr. Denning's identification for spot (C) $R = 10^h\ 38^m$ we have the following as given in his table :

Date 1903.	Obs.	Days.	O—E. m
July 6	Ho	0	+10·1
18	Ho	12	− 5·6
29	β	23	−14·4
Aug. 14	β	43	+ 9·9

We have here a range of $24^m\cdot 5$ in the residuals. Such discordant observations would not be good for eye estimates, and certainly have no connection whatever with micrometer work. These observations obviously do not refer to the same object. Mr. Denning seems to place implicit confidence in Barnard's four observations covering a period of 21 days. On the contrary, I have great faith in his micrometer measure of June 23, which is in harmony with Burnham and myself, although it differs 6·8 minutes from his estimate.

Using Barnard's micrometer measure for June 23, from his observations we get the following, $R = 10^h\ 38^m\ 16^s$:

Date.	Days.	O—E. m	Date.	Days.	O—E. m
June 23	0	+0·0	July 13	20	+1·1
24	1	+6·6	14	21	+0·0

which is in confirmation of my value for the rotation period.

I am somewhat surprised that such an experienced observer as Mr. Denning, who has done so much good work on the planet *Jupiter*, should raise the question of the rotation period of the planet *Saturn* with the kind of material at his disposal.

The Barnard spot was observed on five nights by two or more observers.

The following is the range in the time of passage over the central meridian :

Date 1903.	Range. m	Date 1903.	Range. m
June 26	19	July 24	17
July 11	25	Aug. 5	20?
12	20?		

These figures imply an uncertainty of about ten minutes in the time of passage over the central meridian. During this period the spot was a conspicuous object.

In 1878, before the rotation period of the red spot on *Jupiter* was ascertained we had very much the same kind of uncertain observations.

Mr. Denning's suggestion that discordant observations may be due to oscillation in position of the spots on *Jupiter* and *Saturn* is not sound. There is no proof that any spot on *Jupiter* has changed the direction of its motion in a short interval of time. The smallest spot observed has an area of more than one million square miles, and those usually observed many times that area. Presumably there is mass, and according to our ordinary conception of matter in motion we should not expect any sudden stoppage or reversal of motion.

As between micrometer measures on a luminous disc and eye estimates, at this late day, there ought to be no difference of opinion.

The micrometer method for ascertaining the passage of a spot over the central meridian of *Jupiter* was used by Airy in 1835. At that time the chronograph was unknown and the parallel wire micrometer little used. Airy located the spot by transits of the limbs of the planet and spot by the eye and ear method as in meridian observations.

In 1879 I made a few observations of the red spot in the same way, using the chronograph.

I found, however, that much time was saved and greater precision secured by making the measures directly with the micrometer.

It is to be hoped that if any definite spots are seen on *Saturn* at the coming opposition, astronomers who have micrometers will use them, and determine both longitude and latitude at each observation. Two sets of measures of three bisections of the spot and the limbs of the planet, with the corresponding times when the spot is near the central meridian, will give the time of passage over the central meridian by simple proportion.

During the fifteen minutes employed by the observer in watching a spot in transit one may make from twenty to forty micrometer readings.

When observers on the same night differ in the estimated time of passage fifteen or twenty minutes, it is evident that the method is not a good one. Observers who have no driving clock or micrometer must necessarily do the best they can, but it ought to be evident to every working astronomer that the quality of the work cannot be the same for all instruments.

Dearborn Observatory of North-Western University:
1904 *April* 2.

Corrected Continuation of Brünnow's "Tafeln der Flora."

By A. M. W. Downing, D.Sc., F.R.S.

The materials collected in this paper are intended for the convenience of astronomers who may wish to use the corrected elements of *Flora*, published in *Monthly Notices*, vol. lii. p. 590, in combination with Brünnow's Tables, for the purpose of computing an ephemeris. By means of the continuations or modifications of certain of the tables, or of parts of them, here given, the necessary computations can be made with facility for the period 1904–1911.

From an observation of the planet made by Mr. Tebbutt at Windsor, N.S.W., on 1902 July 27, and published in *Astron. Nachrichten*, No. 3856, I find the correction to the place of *Flora*, computed in the manner indicated above, to be

in R.A. $-3^{s}\cdot85$, and in Decl. $-13''\cdot3$.

Continuation of Brünnow's Tafel I., "Argumente für die Jahre."

	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.	XIV.
B. 1904	100·83	13·06	113·90	214·74	188·63	87·8	126·9	289·4	315·6	285·3	298·4	140·0	276·4	311·3
1905	180·65	342·73	163·39	344·05	18·59	197·9	146·1	199·2	164·7	144 8	127·6	128·9	216·5	110·2
1906	260·47	312·40	212·88	113·36	208·55	165·2	308·1	108·9	13·8	4·4	316·8	117·7	156·6	269·1
1907	340·29	282·08	262·38	242·67	38·51	58·2	184·4	18·7	223·0	223·9	146·0	106·6	96·7	68·0
B. 1908	60·33	251·67	312·01	12·34	228·99	168·7	203·6	289·3	72·6	83·1	334·8	95·4	37·6	226·3
1909	140·15	221·34	1·50	141·65	58·95	278·8	222·8	199·1	281·7	302·6	164·0	84·3	337·8	25·2
1910	219·97	191·02	50·99	270·96	248·91	28·9	242·0	108·9	130·9	162·1	353·2	73·1	277·9	184·1
1911	299·79	160·70	100·49	40·28	78·88	139·1	261·2	18·7	340·1	21·6	182·3	61·9	218·0	343·0

	XV.	XVI.	XVII.	XVIII.	XIX.	XX.	XXI.	XXII.	XXIII.	XXIV.	XXV.	XXVI.	XXVII.	XXVIII.
B. 1904	30·4	131·1	56·5	328·6	4·2	118·2	224·0	136·2	0·3	136·6	224·4	311·8	48·4	175·9
1905	20·0	200·5	345·5	147·4	54·4	217·9	321·9	124·0	86·0	210·1	48·0	159·9	286·0	121·9
1906	9·6	270·0	274·4	326·2	104·7	317·7	59·9	111·8	171·7	283·6	231·7	8·0	163·6	67·9
1907	359·2	339·4	203·4	145·0	154·9	57·4	157·8	99·6	257·4	357·1	55·3	216·1	41·2	13·9
B. 1908	349·8	49·9	133·2	324·3	206·2	158·5	255·9	87·3	343·5	70·8	239·5	64·6	278·5	320·8
1909	339·3	119·3	62·1	143·1	256·5	258·2	353·8	75·1	69·2	144·3	63·1	272·7	156·2	266·8
1910	328·9	188·8	351·0	321·9	306·8	357·9	91·8	62·9	154·9	217·8	246·8	120·8	33·9	212·7
1911	318·5	258·2	279·9	140·7	357·1	97·5	189·8	50·7	240·5	291·3	70·4	328·9	271·6	158·7

In the formation of these arguments the mean anomalies of *Flora* have been computed from the corrected elements. The mean anomalies of *Jupiter* and *Saturn* have been taken from Le Verrier's Tables, and are corrected for the great inequality.

Table to be substituted for the First Part (p. 6) of Brünnow's Tafel IV., "Für die mittlere Anomalie."

	M.	*t.*
B. 1904	87° 47′ 23″·68	+55·998
1905	197 55 57·11	+56·997
1906	308 4 30·54	+57·997
1907	58 13 3·97	+58·996
B. 1908	168 39 43·74	+59·998
1909	278 48 17·17	+60·997
1910	28 56 50·60	+61·997
1911	139 5 24·03	+62·996

	M	*t.*
January ...	0° 0′ 0″·00	0·000
February ...	9 21 16·48	+0·085
March ...	17 48 13·95	+0·162
April	27 9 30·43	+0·246
May	36 12 40·58	+0·329
June	45 33 57·06	+0·413
July	54 37 7·21	+0·496
August ...	63 58 23·69	+0·580
September ...	73 19 40·17	+0·665
October ...	82 22 50·32	+0·747
November ...	91 44 6·80	+0·832
December ...	100 47 16·95	+0·915

Days.	M.	*t.*
1	0° 18′ 6″·34	+0·003
2	0 36 12·68	+0·005
3	0 54 19·01	+0·008
4	1 12 25·35	+0·011
5	1 30 31·69	+0·014
6	1 48 38·03	+0·016
7	2 6 44·37	+0·019
8	2 24 50·71	+0·022
9	2 42 57·04	+0·025
10	3 1 3·38	+0·027
20	6 2 6·76	+0·055
30	9 3 10·14	+0·082

In bissextile years (marked B above) subtract one day from the date during January and February.

Corrections to Brünnow's Tafel V., "Für die Mittelpunkts Gleichung," and Tafel VI., "Für den elliptischen Radius Vector."

M.	$\Delta(v-M)$.	$\Delta \log r$.	M.	$\Delta(v-M)$.	$\Delta \log r$
°	"		°	"	
0	0·00	+27	90	−3·50	−29
1	−0·06	+27	95	−3·42	−32
5	−0·49	+26	100	−3·33	−34
10	−0·97	+25	105	−3·17	−37
15	−1·42	+23	110	−2·97	−40
20	−1·85	+21	115	−2·77	−40
25	−2·24	+18	120	−2·57	−40
30	−2·59	+15	125	−2·37	−41
35	−2·89	+11	130	−2·17	−43
40	−3·15	+ 7	135	−1·96	−46
45	−3·38	+ 3	140	−1·75	−49
50	−3·60	− 2	145	−1·54	−49
55	−3·70	− 5	150	−1·33	−49
60	−3·73	− 8	155	−1·12	−49
65	−3·74	−11	160	−0·91	−50
70	−3·75	−15	165	−0·70	−50
75	−3·69	−20	170	−0·48	−50
80	−3·63	−24	175	−0·25	−51
85	−3·57	−26	180	0·00	−51

These corrections reduce the equation of the centre and the log radius vector, taken from Brünnow's Tables, to the corresponding values deduced from the corrected elements.

The corrections for intermediate values of M can be found with sufficient accuracy by interpolation.

When M is greater than 180° the argument is 360°−M. In this case the sign of $\Delta(v-M)$ must be changed, but the sign of $\Delta \log r$ remains unchanged.

The values of $\Delta \log r$ are expressed in units of the seventh decimal.

Continuation of Brünnow's Tafel XIV., "Für die Constanten in Bezug auf den Aequator."

	A.	B.	C.	log *a*.	log *b*.	log *c*.	log cos *a*.	log cos *b*.	log cos *c*.
1904	123° 38′ 45″·49	35° 47′ 9″·86	19° 53′ 6″·24	9·9980036	9·9694658	9·5735743	8·98075	9·55892*n*	9·96717
1905	123 39 35·90	35 47 58·99	19 53 57·99	9·9980040	9·9694696	9·5735477	8·98071	9·55890*n*	9·96717
1906	123 40 26·30	35 48 45·12	19 54 49·74	9·9980044	9·9694734	9·5735211	8·98067	9·55887*n*	9·96717
1907	123 41 16·71	35 49 37·25	19 55 41·50	9·9980047	9·9694772	9·5754945	8·98063	9·55885*n*	9·96718
1908	123 42 7·25	35 50 26·52	19 56 33·40	9·9980051	9·9694811	9·5734678	8·98058	9·55882*n*	9·96718
1909	123 42 57·65	35 51 15·65	19 57 25·17	9·9980055	9·9694849	9·5734412	8·98054	9·55880*n*	9·96718
1910	123 43 48·06	35 52 4·79	19 58 16·94	9·9980059	9·9694887	9·5734145	8·98050	9·55878*n*	9·96719
1911	123 44 38·46	35 52 53·93	19 59 8·71	9·9980063	9·9694925	9·5733878	8·98046	9·55876*n*	9·96719

These quantities have been computed from the corrected elements, and the values of A, B, and C modified by the subtraction of a constant (9′ 46″·5) from each, so as to be applicable to the perturbed true anomalies when the perturbations are taken from Brünnow's Tables.

H.M. Nautical Almanac Office:
1904 *March* 31.

Observations of the Minor Planet (324) *Bamberga at Windsor, New South Wales.* By John Tebbutt.

The observations were made with the 8-inch equatorial and its filar micrometer in a bright field. The following notes were made with reference to the planet's magnitude:

August 12: Magnitude about 8·0. August 17: Apparently brighter than on previous occasions. August 20: Certainly did not exceed $8\frac{1}{2}$ magnitude. August 29: Planet and comparison star exactly equal. September 1: Much brighter than comparison star and of $8\frac{1}{2}$ magnitude. September 3: Planet slightly brighter than comparison star. September 4: Considerably less than star. September 5: Very much inferior to star. September 7: Planet becoming fainter. September 14: Slightly less than star. September 21: Equal to $8\frac{1}{2}$ magnitude.

(324) *Bamberga.*

Date.	Windsor Mean Time	Planet—Star. Δ R.A.	Δ N.P.D.	No. of Comps.	Planet's App. R.A.	Log. $p\Delta$.	Planet's App. N.P.D.	Log. $p\Delta$.	Reduction to App. Place. R.A.	N.P.D.	Comp. Star.
1903.	h m s	m s	′ ″		h m s	−	° ′ ″	+	s	″	
Aug. 12	10 51 56	+1 48·61	− 5 33·5	15	22 23 24·48	9·4355	101 37 19·2	0·5448	+3·34	−23·7	1
13	9 45 47	+0 58·36	−11 0·4	15	22 22 34·25	9·5725	101 31 52·3	0·5767	+3·36	−23·7	1
15	10 26 1	−4 51·32	...	5	22 20 42·88	9·4666	...	...	+3·37	...	2
15	10 26 1	−5 53·16	− 3 28·3	5	22 20 42·93	9·4666	101 20 17·7	0·5542	+3·36	−23·8	3
16	9 33 2	−5 45·97	+ 4 42·4	7	22 19 48·24	9·5690	101 14 47·0	0·5789	+3·38	−23·9	2
16	9 33 2	−6 47·82	...	7	22 19 48·29	9·5690	...	...	+3·38	...	3
17	9 21 59	−6 43·36	− 0 59·8	7	22 18 50·86	9·5791	101 9 4·8	0·5835	+3·39	−23·9	2
17	9 21 59	−7 45·29	...	7	22 18 50·83	9·5791	...	...	+3·39	...	3
20	9 32 58	−3 12·77	+10 48·0	12	22 15 50·48	9·5318	100 51 40·5	0·5736	+3·44	−24·0	4
22	9 15 26	−5 16·08	− 0 47·9	10	22 13 47·18	9·5464	100 40 4·5	0·5797	+3·45	−24·1	4
23	9 16 41	−6 19·02	− 6 40·5	10	22 12 44·25	9·5342	100 34 11·9	0·5781	+3·46	−24·1	4
29	9 13 24	+1 4·54	+ 6 53·5	15	22 6 20·84	9·4729	99 58 18·4	0·5744	+3·51	−24·2	5
Sept. 1	8 55 46	+0 59·78	+ 1 9·9	15	22 3 12·23	9·4794	99 40 5·2	0·5793	+3·52	−24·2	6
2	8 52 43	−0 2·40	− 5 1·4	7	22 2 10·06	9·4744	99 33 53·8	0·5800	+3·53	−24·3	6
3	8 26 24	−1 2·78	−11 8·7	15	22 1 9·68	9·5235	99 27 46·5	0·5895	+3·53	−24·3	6
4	8 33 19	+0 41·84	+10 43·5	15	22 0 8·61	9·4972	99 21 29·0	0·5863	+3·52	−24·3	7

Date.	Windsor Mean Time.	Planet—Star. Δ R.A.	Δ N.P.D.	No. of Comps.	Planet's App. R.A.	Log. $p\Delta$.	Planet's App. N.P.D.	Log. $p\Delta$.	Reduction to App. Place. R.A.	Reduction to App. Place. N.P.D.	Comp. Star.
1903.	h m s	m s	′ ″		h m s	−	° ′ ″	+	s	″	
Sept. 5	8 30 35	−0 17·92	+ 4 25·8	15	21 59 8·85	9·4919	99 15 11·3	0·5868	+3·52	−24·3	7
7	8 23 35	−2 14·42	− 8 16·0	10	21 57 12·35	9·4851	99 2 29·5	0·5885	+3·52	−24·3	7
7	9 10 54	−2 16·36	...	5	21 57 10·41	9·3400	...	...	+3·52	...	7
7	9 10 54	+4 36·14	+ 1 5·0	5	21 57 10·34	9·3400	99 2 14·8	0·5740	+3·52	−24·2	8
8	7 44 48	−3 9·42	...	7	21 56 17·35	9·5560	...	...	+3·52	...	7
8	7 44 48	+3 43·07	− 4 55·9	7	21 56 17·27	9·5560	98 56 13·9	0·6022	+3·52	−24·2	8
9	8 6 35	−4 5·66	...	3	21 55 21·11	9·5019	...	...	+3·52	...	7
9	8 6 35	+2 46·73	−11 30·0	3	21 55 20·93	9·5019	98 49 39·8	0·5934	+3·52	−24·2	8
12	8 58 31	+6 43·60	+ 8 36·4	6	21 52 41·70	9·2911	98 29 54·6	0·5790	+3·50	−24·2	9
13	8 3 21	+5 56·32	+ 2 16·5	10	21 51 54·42	9·4624	98 23 34·6	0·5936	+3·50	−24·3	9
14	7 59 33	+5 8·88	− 4 19·9	10	21 51 6·97	9·4600	98 16 58·2	0·5950	+3·49	−24·3	9
20	8 3 33	−4 9·15	+10 28·7	8	21 47 1·26	9·3612	97 36 29·1	0·5957	+3·46	−24·5	10
21	7 58 38	−4 42·80	+ 3 37·4	7	21 46 27·60	9·3626	97 29 37·8	0·5972	+3·45	−24·5	10
23	7 48 59	−5 42·78	−10 11·3	7	21 45 27·61	9·3652	97 15 49·2	0·6005	+3·44	−24·4	10

Adopted Mean Places of the Comparison Star for 1903·0.

Star.	R.A. h m s	N.P.D. ° ′ ″	Authorities.
1	22 21 32·53	101 43 16·4	Argent, G. Cat. 1875, 30622; Radcliffe, 1890, 6026.
2	22 25 30·83	101 10 28·5	Greenw. Cat. 1880, 3769; Stone, 11769; Radcliffe, 1890, 6040.
3	22 26 32·73	101 24 9·8	Greenw. Cat. 1880, 3778; Stone, 11780; Radcliffe, 1890, 6043.
4	22 18 59·81	100 41 16·5	Argent, G. Cat. 1875, 30577; Radcliffe, 1890, 6012.
5	22 5 12·79	99 51 49·1	Lalande, 43217 and 8.
6	22 2 8·93	99 39 19·5	Yarnall$_3$, 9955.
7	21 59 23·25	99 11 9·8	Radcliffe, 1890, 5935.
8	21 52 30·68	99 1 34·0	Radcliffe, 1890, 5911.
9	21 45 54·60	98 21 42·4	Radcliffe, 1890, 5889.
10	21 51 6·95	97 26 24·9	Radcliffe, 1890, 5906.

Private Observatory, The Peninsula, Windsor, N.S. Wales:
1904 *February* 11.